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Yu.V. Svishchov

O.Ya. Usikov Institute for Radiophysics and Electronics of NASU
12, Akad. Proskura St., Kharkiv, 61085, Ukraine
E-mail: YuSvishchov@gmail.com

On some behavioral peculiarities of magnetic-type eigenmodes of a spherical particle with arbitrarily valued material parameters

Subject and Purpose. *The spectral characteristics (eigenfrequencies, eigenmodes, Q-factors) of a spherical particle with arbitrarily valued permittivity and permeability are considered to take a further look into some important features of their behavior. The real and imaginary parts of the material parameters of the particle can be both positive and negative. The emphasis is on magnetic-type modes.*

Methods and Methodology. *The spectral problem is solved using the electromagnetic field expansion in vector spherical wave functions.*

Results. *The first eigenfrequencies of a spherical particle have been calculated depending on its relative permittivity ε_1 and relative permeability μ_1 whose real and imaginary parts can take both positive and negative values. The eigenmodes split into two, internal and external, eigenmode families. The internal eigenmodes bear an independent, associated with eigenmode structure labeling in each quadrant of the plane (μ_1, ε_1). The external eigenmodes, on the contrary, have a uniform labeling throughout the whole (μ_1, ε_1) plane and bear a structural resemblance to surface plasmon oscillations distributed in the vicinity of the particle surface or outside it. In the first quadrant of the plane (μ_1, ε_1), the external eigenmodes repeatedly interact with the internal eigenmodes, leading to either mode hybridization or mode type exchange. In the third quadrant of the plane (μ_1, ε_1), the external eigenmodes can interact with one another. The anomalous behavior of the spectral characteristics of a spherical particle corresponds to the already known phenomenon of wave mode coupling described in the scientific literature well enough.*

Conclusion. *The performed study has revealed some new behavioral patterns as to the spectral characteristics of a spherical particle with arbitrarily valued permittivity and permeability. Fig. 5. Ref.: 13 items.*

Key words: *spherical particle, dielectric ball, metamaterial, eigenfrequencies, eigenmodes.*

The study of spherical particles with arbitrarily valued material parameters is facilitated by the fact that any formula obtained for an ordinary matter can be applied to the case when permittivity and permeability can take arbitrary values. The eigenfrequency equations of spherical dielectric resonators were first obtained by Mie and Debye in 1908–1909 [1, 2]. Over a wide permittivity range, numerical calculations of complex-valued eigenfrequencies were first performed in [3]. An excellent eigenmode analysis of an isotropic spherical dielectric resonator was carried out in [4]. The electrodynamic properties of a spherical particle with arbitrary real-valued material parameters were first considered in [5]. That time the particle resonant properties were studied by analysis of reflection coefficient resonances at a real-valued wave number. In [4, 5], it was noted, in particular, that in the particle of the kind, there are modes demonstrating abnormal (i.e. qualitatively different from normal or ordinary) behavior of the spectral curves. The described in [4, 5] abnormal behavior of electric-type mode spectral curves was thoroughly studied in [6] for a dielectric ball and in [7] for a spherical particle with simultaneously negative permittivity and permeability. The spectral curves reported in [4–7] belong to the spectral curve class of electrodynamic structures under the

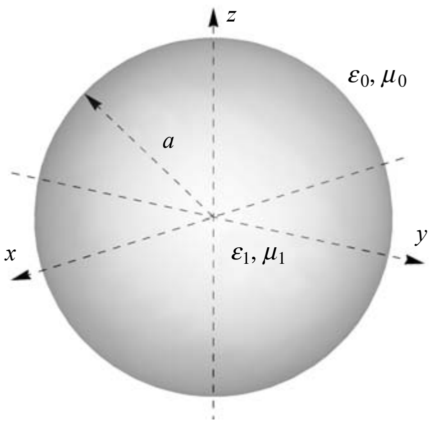


Fig. 1. Spherical particle

wave mode coupling conditions [8]. This circumstance is a driving factor to take a further look into the resonant properties of a spherical particle by analysis of its spectral characteristics in the complex frequency range.

The present work is concerned with behavioral patterns of spectral characteristics of a spherical particle having arbitrarily valued permittivity and permeability. The emphasis is on magnetic-type modes.

No restrictions are imposed on the permittivity and permeability, enabling simultaneously different signs of their real and imaginary parts. Correspondingly, a question arises as to the refractive index sign choice for a medium with complex-valued material parameters. A complete analysis of refractive index signs for loss and gain media can be found in [9] (see, also, [10]). In the present problem solution, a simple formula from [11] is adopted. The final results coincide with the data obtained by other algorithms of the refractive index calculation [10] (in view of the active media comments in [10]).

Mathematical problems arise in the refractive index sign determination and refer to the question of complex square root calculation because the complex plane can be cut into two Riemannian surface sheets in different ways, each suitable for a certain medium type. In the general case, however, it is hard to choose one. The mentioned ambiguity prevents us from linking a proper value of the refractive index to one of the two branches of the complex square root function. Therefore the refractive index sign determination should be performed based on some additional physical considerations. Normally, refractive index sign is sought in conjunction with complex impedance sign. A condition is proposed [11] that the sign of the refractive index imaginary part and the sign of the loss or gain energy density be coincident.

1. Calculation method. Let us consider a spherical particle of radius a (Fig. 1), relative permittivity $\varepsilon_1 = \varepsilon_1' + i\varepsilon_1''$ and relative permeability $\mu_1 = \mu_1' + i\mu_1''$. The relative permittivity and the relative permeability of the surrounding space are, respectively, $\varepsilon_0 > 0$ and $\mu_0 > 0$. The wave number inside and outside the particle is $k_s = k\sqrt{\varepsilon_s \mu_s}$, $s = 0,1$, where $k = \omega / c$ is the wave number in vacuum. A spherical coordinate system (r, θ, φ) with the origin at the center of the particle is introduced, where $\vec{r}$ is the radius-vector of the observation point. The electromagnetic field inside and outside the particle (in the incident field absence) is sought as

$$(\vec{E}, \vec{H}) = \begin{cases} (\vec{E}^{\,0}, \vec{H}^{\,0}), & r > a, \\ (\vec{E}^{\,1}, \vec{H}^{\,1}), & r < a, \end{cases} \tag{1}$$

where

$$\begin{aligned} \vec{E}^0(\vec{r}) &= \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \bar{D}_{mn}^{0} \vec{M}_{mn}^{(3)}(\vec{r}, k_0) = \\ &= \sum_{m=0}^{\infty} \sum_{n=(m,1)}^{\infty} \left\{ \begin{matrix} \bar{D}_{mn}^{0(+)} \vec{M}_{mn}^{e(3)}(\vec{r}, k_0) + \\ + i \bar{D}_{mn}^{0(-)} \vec{M}_{mn}^{o(3)}(\vec{r}, k_0) \end{matrix} \right\}, \\ \vec{H}^0(\vec{r}) &= \frac{k_0}{ik\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \bar{D}_{mn}^{0} \vec{N}_{mn}^{(3)}(\vec{r}, k_0) = \\ &= \sum_{m=0}^{\infty} \sum_{n=(m,1)}^{\infty} \left\{ \begin{matrix} \bar{D}_{mn}^{0(+)} \vec{N}_{mn}^{e(3)}(\vec{r}, k_0) + \\ + i \bar{D}_{mn}^{0(-)} \vec{N}_{mn}^{o(3)}(\vec{r}, k_0) \end{matrix} \right\}; \end{aligned} \tag{2}$$

$$\begin{aligned} \vec{E}^1(\vec{r}) &= \sum_{n=1}^{\infty} \sum_{m=-n}^{n} D_{mn}^{1} \vec{M}_{mn}^{(1)}(\vec{r}, k_1) = \\ &= \sum_{m=0}^{\infty} \sum_{n=(m,1)}^{\infty} \left\{ \begin{matrix} D_{mn}^{1(+)} \vec{M}_{mn}^{e(1)}(\vec{r}, k_1) + \\ + i D_{mn}^{1(-)} \vec{M}_{mn}^{o(1)}(\vec{r}, k_1) \end{matrix} \right\}, \\ \vec{H}^1(\vec{r}) &= \frac{k_1}{ik\mu_1} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} D_{mn}^{1} \vec{N}_{mn}^{(1)}(\vec{r}, k_1) = \\ &= \frac{k_1}{ik\mu_1} \sum_{m=0}^{\infty} \sum_{n=(m,1)}^{\infty} \left\{ \begin{matrix} D_{mn}^{1(+)} \vec{N}_{mn}^{e(1)}(\vec{r}, k_1) + \\ + i D_{mn}^{1(-)} \vec{N}_{mn}^{o(1)}(\vec{r}, k_1) \end{matrix} \right\}; \end{aligned} \tag{3}$$

$h_{mn}^{(\pm)} \overset{def}{=} \left[h_{mn} \pm (-1)^m h_{-mn} \right] \left[1 + \delta_{m0} \right]^{-1}$, $\quad (m,1) \overset{def}{=}$ $= \max(m,1)$, and $\bar{D}_{mn}^{0}$ and D_{mn}^{1} are the unknown coefficients. The vector spherical wave functions $\vec{N}_{mn}^{(l)}(\vec{r},k_s)$ and $\vec{M}_{mn}^{(l)}(\vec{r},k_s)$ and the even and odd vector spherical wave functions $\vec{N}_{mn}^{e(l)}(\vec{r},k_s)$, $\vec{N}_{mn}^{o(l)}(\vec{r},k_s)$ and $\vec{M}_{mn}^{e(l)}(\vec{r},k_s)$, $\vec{M}_{mn}^{o(l)}(\vec{r},k_s)$ [12, 13] entering (1)–(3) accurate to the normalization factors adopted in the work are defined in [6, 7]. From here on, the time dependence of the fields is assumed to be $\exp(-i\omega t)$.

With the boundary condition that the tangential components of the $\vec{E}$ and $\vec{H}$ fields are continuous on the particle surface ($r = a$), we obtain the dispersion equation of the magnetic-type eigenmodes (the *TE* or *H* modes)

$$\frac{k_0}{k_1}\psi_n(k_1 a)\zeta_n'(k_0 a) - \frac{\mu_0}{\mu_1}\psi_n'(k_1 a)\zeta_n(k_0 a) = 0, \quad (4)$$

where $\psi_n(z) = \sqrt{\frac{\pi z}{2}} J_{n+1/2}(z)$ and $\zeta_n(z) = \sqrt{\frac{\pi z}{2}} H_{n+1/2}^{(1)}(z)$ are the Riccati-Bessel functions of the first and third kind and $J_{n+1/2}(z)$ and $H_{n+1/2}^{(1)}(z)$ are the Bessel and Hankel functions of a half-integer index. Assuming D_{mn}^{1} known, we find

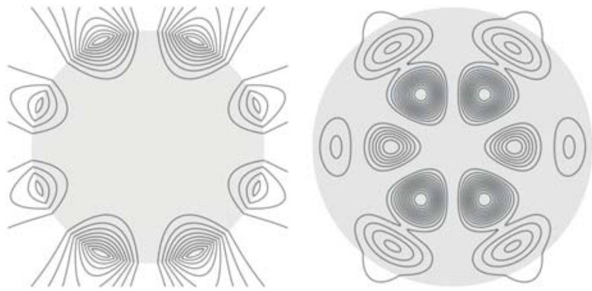
$$\bar{D}_{mn}^{0} = \frac{k_0}{k_1}\frac{\psi_n(k_1 a)}{\zeta_n(k_0 a)} D_{mn}^{1}.$$

Care should be taken of the following points. First, the natural area of the analytical continuation of the boundary value problem solution in wave number is the complex plane except for $k = 0$ (Ricatti-Bessel functions are analytic in this area). Second, as the arguments of the functions in (4) involve $\sqrt{\varepsilon_1\mu_1}$, a question of proper choice of the root sign arises. We will calculate $\sqrt{\varepsilon_1\mu_1}$ by the formula [11] (see above)

$$\sqrt{\varepsilon_1\mu_1} = \sqrt{\varepsilon_1}\,\sqrt{\mu_1}\,\mathrm{sign}\left[\mathrm{Re}\left(\frac{\sqrt{\varepsilon_1}}{\sqrt{\mu_1}}\right)\right],$$

where the square roots are considered in the principal value sense when the cut is along the negative half-axis of the real axis ($\sqrt{z} \ge 0$, z is a non-negative real number). This principal value selection is used to advantage in the most important mathematical Application Packages.

2. Brief eigenmode analysis. A distinctive feature of magnetic-type modes is a nonzero radial component H_r of the magnetic field ($E_r = 0$). Accordingly, magnetic type modes are described (see (1)–(3)) by the vectors $\vec{M}_{mn}^{(l)}(\vec{r},k_s)$ and $\vec{N}_{mn}^{(l)}(\vec{r},k_s)$ (the radial component present). The eigenfrequencies of these modes come from (4). That the azimuthal index m does not enter equation (4) means the eigenmode degeneracy with respect to m. The degeneracy rate is $2n+1$, which follows from (2), (3) in terms of the even and odd vector spherical wave functions. For the *TE*-modes, the modes TE_{mnq}^{e} (m = 0, …, n) described by the fields $\vec{E}_{mnq}^{s}(\vec{r}) = B_{mn}^{s(+)}\vec{M}_{mn}^{e(l)}$ and $\vec{H}_{mnq}^{s}(\vec{r}) = \frac{k_s}{ik\mu_s} B_{mn}^{s(+)}\vec{N}_{mn}^{e(l)}$ and the modes TE_{mnq}^{o} ($m = 1, \ldots, n$) given by the fields $\vec{E}_{mnq}^{s}(\vec{r}) = iB_{mn}^{s(-)}\vec{M}_{mn}^{o(l)}$ and $\vec{H}_{mnq}^{s}(\vec{r}) = \frac{k_s}{ik\mu_s} iB_{mn}^{s(-)}\vec{N}_{mn}^{o(l)}$ ($B_{mn}^{1(\pm)} = D_{mn}^{1(\pm)}$ and $B_{mn}^{0(\pm)} = \bar{D}_{mn}^{0(\pm)}$) correspond to the normalized eigenfrequency $k_{nq}a = \frac{\omega_{nq}}{c}a$, where q is the root number of equation (4). For certainty, the eigenmodes TE_{0nq} (TE_{0nq}^{e}) will be analyzed. The TE_{0nq} modes have the three components E_{φ}, H_r, and H_{θ}, with H_r and H_{θ} calculated through the only electric field component E_{φ}. Hence, while on the subject of the TE_{0nq} mode structure, the spatial distribution of the E_{φ} component is meant.



a b

Fig. 2. The $|\vec{E}|$ distribution for the eigenmodes: a – $TE_{041'}$ ($\mu_1 = -2$, $\varepsilon_1 = -1$); b – TE_{032} ($\mu_1 = 5$, $\varepsilon_1 = 1$)

The eigenmode spectrum of a spherical particle splits into internal (the field inside the particle) and external (the field in the vicinity of the particle surface and outside it) modes (Fig. 2). It consists of two families determined by different dependences on the particle permittivity. So, the roots of the characteristic equations of the internal and external modes are suitable to label differently. To avoid confusion, we leave the labeling TE_{mnq} to the inter-

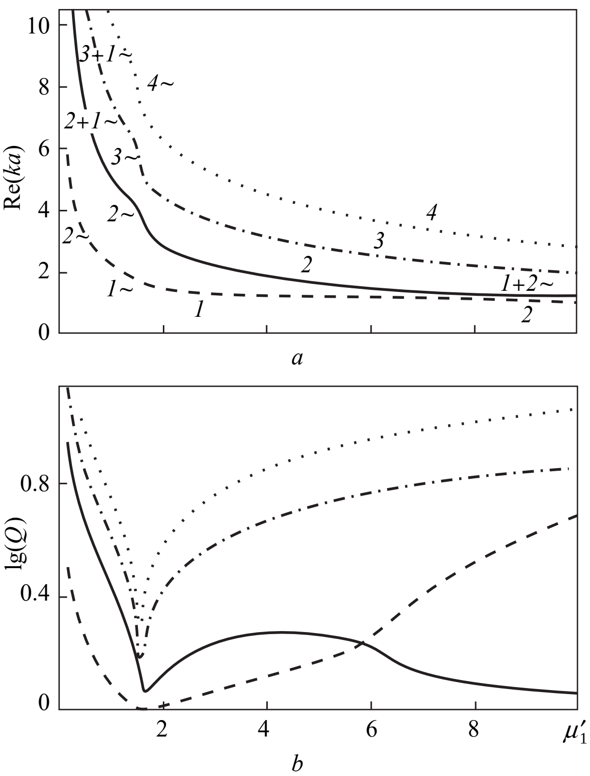


Fig. 3. Re(ka) (a) and lg(Q) (b) depending on μ_1' for the eigenmodes $TE_{011'}$ (*1*), TE_{011} (*2*), TE_{012} (*3*), and TE_{013} (*4*) ($\varepsilon_1' = 1.5$, $\varepsilon_1'' = \mu_1'' = 0$)

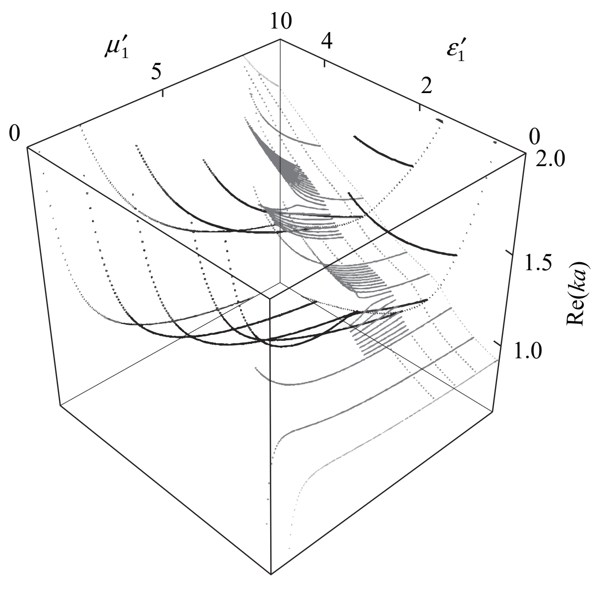


Fig. 4. Re(ka) depending on μ_1' and ε_1' for the eigenmodes $TE_{011'}$ (gray colored), TE_{011} (black bottom surface) and TE_{012} (black top surface) ($\mu_1'' = \varepsilon_1'' = 0$). The surface section with the plane $\varepsilon_1' = 1.5$ is shown in Fig. 3, the eigenmode type indicated

nal modes and attach the labeling $TE_{mnq'}$ with index q primed to the external modes. For the axially symmetric modes $TE_{0nq'}$, index n denotes the number of $|\vec{E}|$ antinodes along the coordinate θ in the spherical coordinate system, and index q' denotes the root number of equation (4). For the internal eigenmodes TE_{0nq}, index q denotes the number of $|\vec{E}|$ antinodes along the coordinate r in the spherical coordinate system. In the present work (unlike [3, 4]), the mode labeling is defined by the electromagnetic field structure rather than the eigenfrequency real part increase. The latter is acceptable when the dependences of the eigenfrequency real parts do not meet. The grounds for the classification by electromagnetic field structure are given by the eigenmode wave coupling described in [6, 7].

3. Behavioral peculiarities of spectral characteristics. Let us consider the eigenfrequency behavior in the first quadrant of the plane (μ_1', ε_1') with $\mu_1'' = \varepsilon_1'' = 0$ ($\mu_0 = \varepsilon_0 = 1$). Fig. 3 presents the (normalized) eigenfrequency real parts Re(ka) and the Q-factor logarithms lg(Q) ($Q = -0.5\mathrm{Re}(ka)/\mathrm{Im}(ka)$) for the eigenmodes $TE_{011'}$, TE_{011}, TE_{012}, and TE_{013} as μ_1' functions at different ε_1' values: $\varepsilon_1' = 1.0$ (a), 1.5 (b), 2.0 (c), 3.0 (d), and 4.0 (e). The symbols like $3+1\sim$ or $3\sim$ mark hybrid-type eigenmodes [6]. The plots are similar to those obtained for electric-type modes in [6]. We claim that the discovered and investigated in [6] phenomenon of wave mode coupling in a dielectric sphere with $\mu_1' = 1$ ($\mu_1'' = \varepsilon_1'' = 0$) takes place for magnetic-type modes, too. The sphere relative permeability μ_1' is a control parameter of this phenomenon (with ε_1' fixed). The external mode $TE_{011'}$ comes into interaction with the internal modes.

The Re(ka) and lg(Q) of the modes $TE_{011'}$, TE_{011}, and TE_{012} are plotted in Fig. 4 as functions of two, μ_1' and ε_1', variables. As seen, in the vicinities of some points of the plane (μ_1', ε_1'), two eigenfrequencies locally form a single double-sheet surface. The cuts of this surface by the $\varepsilon_1' = \mathrm{const}$ plane show that the real and imaginary parts of the eigenmodes behave differently, as described in [6]. In particular, both mode hybridization and mode type exchange are locally seen. Fig. 4 shows multiple mode transformations in the plane (μ_1', ε_1'). It is safe to assume that via a proper μ_1'' and ε_1'' selection, the eigenmode degeneration in the complex frequency domain can be achieved [4].

In Fig. 3, one can also observe lg(Q) resonance minima. As μ_1' μ_0 ($\mu_1' > \mu_0$, $\varepsilon_1' = \varepsilon_0$), the Q-factor of the dielectric ball oscillations tends to zero $(\lg(Q)\to-\infty)$, and the field is ejected out of the resonator as vacuum is approached (the oscillation amplitude therewith tends to zero). As ε_1' increases, this limiting case is drawn away. The field is partially ejected from the dielectric sphere, causing lg(Q) minima. As ε_1' increases, these minima gradually fade. The oscillations in the Q minimum vicinity resemble the $TE_{011'}$ and the TE_{01q} mode structure (depending on the extent to which the antinodes of the original eigenmodes $TE_{011'}$ and TE_{01q} are ejected out of the resonator). In this connection, they are called hybrid modes. When moving away from the lg(Q) minimum towards smaller μ_1' values, hybrid oscillations with the TE_{01q} and $TE_{011'}$ mode features appear, too. These oscillations have antinodes inside the ball and an antinode with a significantly lower amplitude in the ball surface vicinity. These two hybridity types fundamentally differ from the oscillation hybridity under the wave mode coupling condition when hybrid oscillations have features of two oscillations entering wave mode coupling.

Equation (4) finds the eigenfrequencies ka whose real parts give electromagnetic field eigenfrequency values and whose imaginary parts describe attenuation of the modes. Calculations of the eigenfrequencies in the third quadrant of the plane (μ_1', ε_1') show that extra solutions corresponding to electromagnetic field modes not decaying with time are possible. The result is unexpected but it, unfortunately, takes place. In this connection, physically correct solutions are necessary to select based on the condition $\mathrm{Im}(ka) < 0$. The introduction, for example, the $\mu_1'' > 0$ and $\varepsilon_1'' > 0$ values into the calculations only worsens the result. The introduction of $\mu_1'' < 0$ and $\varepsilon_1'' < 0$ shows (Fig. 5) that as $\mu_1' < 0$ increases in absolute value, then, starting with a certain point, the ka imaginary part gets negative, suggesting that the solution becomes physically correct. It should also be noted that the existence of physically incorrect solutions of (4) leads to resonances of the Mie coefficients [5], which enables the author of [5] to draw a conclusion about internal mode existence in the third quadrant of the plane (μ_1', ε_1'). Physically incorrect solutions arise in the first quadrant, too, as soon as $\mu_1'' < 0$ and $\varepsilon_1'' < 0$ are introduced into the calculations: as $\mu_1' > 0$ increases, then starting with some μ_1' value, the ka imaginary part becomes positive.

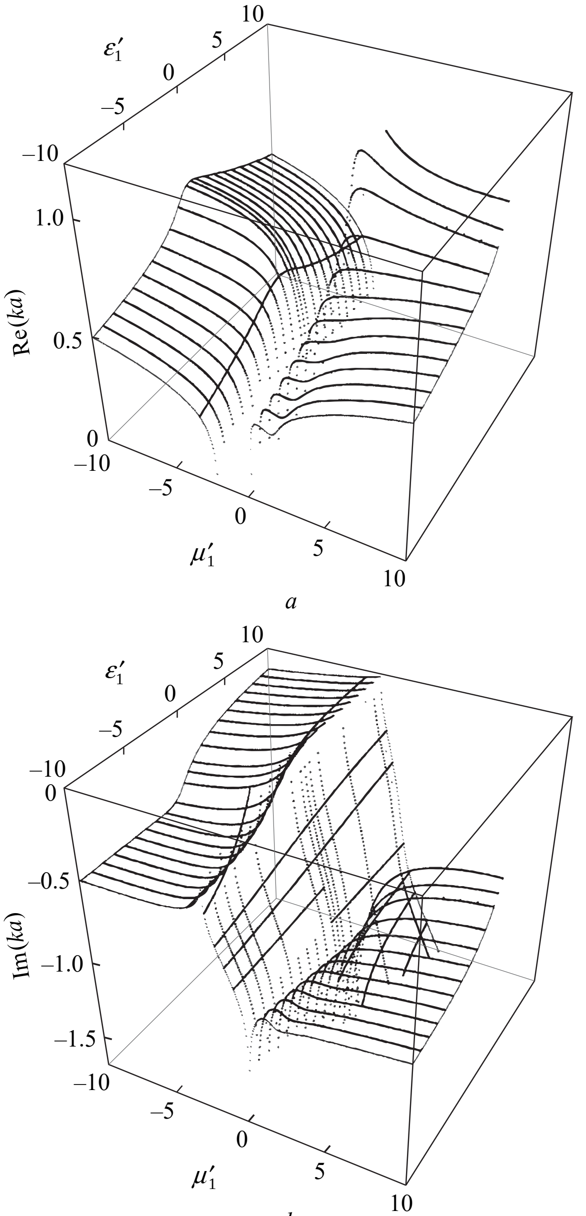


Fig. 5. Re(ka) (a) and Im(ka) (b) depending on μ_1' and ε_1' for the eigenmode $TE_{011'}$ $(\varepsilon_1'' = \mu_1'' = 0)$

The plasmon oscillation $TE_{011'}$ deserves especial attention. Unlike the internal eigenmodes, it occurs in all the quadrants of the plane (μ_1', ε_1'), and its eigenfrequency is characterized by important patterns (Figs. 3–5). Let a point belonging to the first quadrant of the plane (μ_1', ε_1') describe a simple curve starting from the first quadrant and bypassing the origin clockwise. Then as the parameters (μ_1', ε_1') vary along this curve, the eigenfrequency of the eigenmode $TE_{011'}$ at $\varepsilon_1' < \varepsilon_0$, $\mu_1' > \mu_0$ (the first quadrant) first passes to the fourth quadrant, then to the third and second quadrants and, finally, returns to the first quadrant (but now with a zero-valued real part). That is, this eigenmode frequency behaves as if it belonged to some helical surface with its axis at the point (μ_0, ε_0). In this case, during the passage from the fourth to the third quadrant (over some μ_1' variation interval), as well as during the passage from the second to the first quadrant, the eigenfrequency real part vanishes. The eigenfrequency behavior in the first quadrant (at $\varepsilon_1' \geq \varepsilon_0$) was described above (Figs. 3 and 4). During the passage from the first quadrant of the plane (μ_1', ε_1') to the fourth one, the eigenmode $TE_{011'}$ field is increasingly pressed to the surface as ε_1' decreases. During the fourth to the third quadrant passage, the field is completely ejected out of the particle at the point (μ_1'), where the $\mathrm{Im}(ka)$ is at its minimum. With a further μ_1' decrease, the field is restored and has a form of a "classical" plasmon eigenmode whose *Q*-factor increases during the passage to the second quadrant.

We have considered some behavioral peculiarities of the $n = 1$ eigenmodes, the simplest to analyze. As *n* increases, the eigenfrequency and eigenmode behavior can get more complexity (from the visualization standpoint), especially under wave mode coupling conditions. Thus, by analogy with electric-type modes [7], it has been found that the $TE_{031'}$ and $TE_{032'}$ eigenmodes come into interaction in the third quadrant of the plane (μ_1', ε_1'). As a consequence, when the particle material parameters vary in the critical point vicinity [7], either mode hybridization or mode type exchange is locally observed. Unlike the $TE_{011'}$ oscillation, the real part of the plasmon oscillation frequency in the fourth to the third quadrant passage is other than zero, the imaginary part of the frequency therewith has a minimum, as for the $TE_{011'}$ oscillation. In the (μ_0, ε_0) point vicinity, the $TE_{031'}$ oscillation frequency changes into the $TE_{032'}$ oscillation frequency as if it followed a helical path with the axis at the point (μ_0, ε_0).

Conclusions. The spectral problem solution for a spherical particle has been obtained, based on the electromagnetic field expansion in vector spherical wave functions.

The first eigenfrequencies (magnetic-type eigenmodes) of a spherical particle have been calculated depending on its relative permittivity ε_1 and relative permeability μ_1' whose real and imaginary parts can take both positive and negative values. The eigenmodes split into two, internal and external, eigenmode families. The internal eigenmodes in each quadrant of the plane (μ_1, ε_1) have a local, independent labeling based on the oscillation structure. Unlike the internal eigenmodes, the external eigenmodes bear a single labeling throughout the plane (μ_1, ε_1).

The external eigenmodes structurally resemble surface plasmon oscillations. In the first quadrant of the plane (μ_1, ε_1), they repeatedly come into the interaction with the internal eigenmodes, resulting in either eigenmode hybridization or eigenmode type exchange. In the third quadrant of the plane (μ_1, ε_1), the external eigenmodes can interact with one another. The anomalous behavior of the spectral characteristics of a spherical particle corresponds to the wave mode coupling phenomenon well described in the scientific literature.

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Ю.В. Свіщов

Інститут радіофізики та електроніки ім. О.Я. Усикова НАН України
12, вул. Акад. Проскури, Харків, 61085, Україна

ПРО ДЕЯКІ ОСОБЛИВОСТІ ПОВЕДІНКИ
ВЛАСНИХ КОЛИВАНЬ МАГНІТНОГО ТИПУ СФЕРИЧНОЇ ЧАСТИНКИ
З ДОВІЛЬНИМИ ЗНАЧЕННЯМИ МАТЕРІАЛЬНИХ ПАРАМЕТРІВ

Предмет і мета роботи. Розглядається поведінка спектральних характеристик (власних частот, власних коливань, добротностей власних коливань) сферичної частинки з довільними значеннями діелектричної і магнітної проникності. Метою цієї роботи є вивчення деяких важливих закономірностей поведінки спектральних характеристик частики як з додатними, так і від'ємними значеннями дійсних та уявних частин матеріальних параметрів. Акцент робиться на коливаннях магнітного типу.

Методи і методологія роботи. Для досягнення поставленої мети наведено розв'язання відповідної спектральної задачі. Метод розв'язання заснований на зображенні електромагнітного поля у вигляді розкладення за векторними сферичними хвильовими функціями.

Результати роботи. Розраховано залежності перших власних частот сферичної частинки від відносної діелектричної ε_1 і відносної магнітної μ_1 проникностей, дійсні та уявні частини яких можуть набувати як додатних, так і від'ємних значень. Коливання поділяються на два сімейства – внутрішні і зовнішні. Внутрішні коливання в кожному з квадрантів площини (μ_1, ε_1) мають незалежну класифікацію, засновану на структурі коливань. На відміну від внутрішніх, зовнішні коливання мають єдину класифікацію в площині (μ_1, ε_1). За своєю структурою зовнішні коливання мають вигляд поверхневих плазмонних коливань, які розподілені в околі поверхні частинки або поза нею. У першому квадранті площини (μ_1, ε_1) вони багато разів вступають у взаємодію з внутрішніми коливаннями, що приводить або до гібридизації коливань, або до обміну типами коливань. У третьому квадранті площини (μ_1, ε_1) зовнішні коливання можуть вступати у взаємодію одне з одним. Аномальна поведінка спектральних характеристик сферичної частинки відповідає вже відомому і добре описаному в науковій літературі явищу міжтипового зв'язку коливань.

Висновок. Результати проведених досліджень дозволили встановити нові закономірності поведінки спектральних характеристик сферичної частинки з довільними значеннями її діелектричної і магнітної проникності.

Ключові слова: *сферична частинка, діелектрична куля, метаматеріал, власні частоти, власні коливання.*